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Journal of Wind Engineering
and Industrial Aerodynamics 92 (2004) 935–958

JOURNAL OF wind engineering AND industrial aerodynamics

www.elsevier.com/locate/jweia

Aerodynamics of spinning and non-spinning tennis balls

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Received 11 December 2003; received in revised form 20 May 2004; accepted 20 May 2004

Abstract

The aerodynamic properties of a tennis ball are obtained using wind tunnel measurements. In the first phase of this study, the drag coefficient of a variety of new and used non-spinning tennis balls was measured. The measurements were conducted in the Reynolds number range of $85 \times 10^3 < Re < 250 \times 10^3$, which corresponds to a velocity range of $20 < U < 60\,\mathrm{ms}^{-1}$ ($45 < U < 135$ mph). The data for the new tennis balls revealed that all balls have similar drag coefficients, within the repeatability of the experiment. However, it was shown that a heavily worn ball exhibits a significant decrease in drag coefficient. A simple trajectory plot was used to illustrate the significance of this drop in C_D, on a typical tennis stroke. It was shown that if the drag coefficient of the worn ball was reduced even further, then the ball would travel faster through the air, and give the receiver a significantly shorter time to react to the shot.

In the second phase of this study, the drag and lift coefficients of new and used spinning tennis balls was measured. This testing was conducted by placing the ball inside the working section of the wind tunnel, held between two horizontal shafts. The ball was spun at rates up to 2750 rpm, which is representative of a typical top spin shot hit by a professional player. The data showed that the lift coefficient for all the new balls was identical. However, the lift coefficient was generally lower for the heavily worn ball compared with the new ball. A trajectory plot was calculated to illustrate the effect of the lower drag and lift coefficients of the worn ball, compared to the new ball. It was shown that the lift coefficient value has little effect on the reaction time of the receiver. It was also shown that a lower lift coefficient reduces the window of opportunity for the server because the player is less able to utilise the spin to control the shot to ensure that the ball both clears the net and also lands in the service box. This highlights the fact that it may not be in the manufacturers interest to produce a 'faster' tennis ball with a lower drag coefficient, because the ball may also have a lower lift coefficient, thus reducing the players ability to play certain strokes.

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0167-6105/$ - see front matter
doi:10.1016/j.jweia.2004.05.004

Nomenclature

A	projected ball area
C_D	drag coefficient ($= F_D/1/2\rho A U^2$)
C_L	lift coefficient ($= F_L/1/2\rho A U^2$)
d	ball diameter
F_D	drag force on the ball
F_L	lift force on the ball
Re	Reynolds number ($= Ud/v$)
S	spin coefficient ($= V/U$)
U	wind tunnel velocity
V	circumferential velocity at edge of ball
v	kinematic viscosity of air
μ	dynamic viscosity of air
ρ	density of air

1. Introduction

The origins of tennis lie in the 12th century when it was played in the courtyards of the great and the good of Europe and was known as *jeu de paume*. This evolved into the game Royal Tennis or Real Tennis. It remained a game for the rich and elite until 1874 when Major Clopton Wingfield 'invented' the game of lawn tennis, which he called 'sphairistike'. This was helped by the development of the rubber industry in the mid-late 1800s to make the balls.

One of the first recorded observations regarding the aerodynamics of tennis balls was made by Newton [1]. Two hundred years later, Lord Rayleigh [2] used the classical potential flow theory for flow around a cylinder with circulation to describe the irregular flight of a tennis ball. However, despite these interesting observations, significant advances in the research into the mechanics of the game is relatively recent. Indeed, in the first review on sports ball aerodynamics [3], no scientific studies on tennis balls could be identified and so tennis ball aerodynamics were not discussed.

The trajectory of a tennis ball is determined by the gravitational and aerodynamic forces which act on it during its flight. The first published study of the aerodynamic forces which act on the ball was that by Stepanek [4]. In that paper, the lift and drag coefficients of a tennis ball were obtained for a simulated top-spin lob. The aerodynamic forces which act on the tennis ball were determined by launching spinning tennis balls into a wind tunnel air stream. The measured trajectories were used to determine the drag and lift coefficients (C_D and C_L, respectively) for air speeds of up to 28 m/s (63 mph) and spin rates of 3250 rpm. That study concluded that these coefficients were dependent on the spin coefficient, but independent of Reynolds number (Re). Stepanek measured values of between 0.54 and 0.75 for C_D, and between 0.07 and 0.27 for C_L, for spin coefficients which ranged between 0.05 and 0.7. Furthermore, the extrapolated value of C_D for a non-spinning tennis ball was found to be 0.51. Chadwick and Haake [5] mounted tennis balls on a force balance in a wind tunnel, and obtained a C_D of about 0.53 for both standard and over sized tennis balls. They found that C_D was independent of *Re* over the narrow range which was tested ($200 \times 10^3 < Re < 270 \times 10^3$). Chadwick and Haake also showed that the C_D of the tennis ball could be increased by approximately 6% simply by raising (or fluffing) the nap. Similarly, it was shown that the C_D could be reduced by 6% simply by electric shaving the ball. Smoke flow visualisations were used to illustrate that the increase/decrease in drag was due to the increase/decrease in the size of the wake behind the ball.

In a later study, Mehta and Pallis [6] used smoke flow visualisation and drag force measurements to thoroughly explain the aerodynamic properties of a non-spinning tennis ball. They reported that the C_D of new tennis balls was between 0.6 and 0.7 for *Re* between 80×10^3 and 300×10^3, and that C_D was reduced at high *Re*, and this was assigned to the nap 'laying down' at the high wind speeds. They also tested a selection of tennis balls which had been used for a number of games, by recreational level players (using two balls at a time). They reported that the drag coefficient was 6% higher (than a new ball) after the ball had been used for six games, but then C_D was 6% lower (than a new ball) after the ball had been used for nine games. Mehta and Pallis explained this by using anecdotal evidence in which players claimed that the ball fluffs up initially upon use, and then after nine games the cover will have worn and thus exhibit a lower value of C_D. The magnitude of the change in C_D is identical to that reported by Chadwick [5] for fluffed up and shaved balls. This comparison has an interesting implication for the game of tennis because it implies that a ball typically exhibits extreme changes in C_D, equal to those of a fluffed or shaved ball, in the first several games of the ball's life.

Mehta and Pallis noted that the drag coefficient of a new tennis ball was higher than that of the standard result for a smooth sphere ($C_D = 0.5$). They illustrated that the flow over a tennis ball must be in the transcritical regime, when C_D is essentially independent of *Re*. The additional drag acting on the tennis ball has been assigned to the pressure drag on the individual fuzz elements of the nap. Clearly, as the ball wears, the number of fuzz elements will decrease and thus the additional drag will reduce. Mehta and Pallis illustrated that the reduction in C_D for worn balls is due to this reduction in fuzz drag.

The preceding paragraphs have shown that C_D of a non-spinning new tennis ball is between 0.53 and 0.70. Some of this difference can be accounted for by the different ranges of *Re* which were used to obtain the data. Furthermore, the different methods used by the authors to measure the ball diameter also lead to disagreement between the measured values of C_D. Also, the sensitivity of the force balance, and the method used to account for the presence of the sting which is supporting the tennis ball, will influence the accuracy of C_D.

Since there are many studies which have thoroughly described flow over a non-spinning new tennis ball, it will not be repeated in the current study. However, there is little published literature on the aerodynamics of spinning tennis balls. Mehta and

Pallis [6] presented a flow visualisation study in which an 280 mm diameter tennis ball was spun in a wind tunnel for a range of *Re* (up to 280×10^3). The size of the ball was increased to enable higher *Re* to be tested, at lower wind speeds. However, the felt was not scaled to match the increased size of the ball. They showed that the boundary layer separates earlier on the side of the ball which is moving upwind, compared to the side which is moving downwind. The resulting asymmetric wake produces a lateral (Magnus) force acting on the ball. They found that this force occurred in the expected direction for the full range of speeds and spins which were tested, which suggested that the transition from laminar to turbulent boundary layer had already occurred on both sides of the ball.

As far as the authors are aware, Stepanek [4] published the only paper presenting quantitative data on the aerodynamic properties of a spinning tennis ball for a relatively low wind speed, for only one type of tennis ball. In this current study, the drag and lift coefficients of spinning tennis balls are obtained for a full range of relevant wind speeds and ball types. This work is sponsored by the International Tennis Federation (ITF), who are the governing body of tennis. The main aim of this study was to benchmark the aerodynamic properties of tennis balls which are available on shop shelves. Furthermore, the ITF intends to further their knowledge of the mechanics of the game by understanding the aerodynamic properties of a wide range of ball types. Currently, the rules of tennis do not specify any technical specifications regarding the outer covering on a tennis ball, as noted by Mehta and Pallis [6]. The ITF rule simply states that, "The ball shall have a uniform outer surface consisting of a fabric cover and shall be white or yellow in colour". The findings of this study will aid the ITF in deciding whether a new ruling, regarding the aerodynamics of the ball surface, is required to protect the nature of the game. This would be implemented to prevent manufacturers from exploiting a possible loop hole in the ball rules, and prevent them from introducing a tennis ball with radically different aerodynamic properties compared to a current tennis ball.

2. Experimental methods

There are two main methods which can be used to measure the aerodynamic forces acting on a spinning tennis ball. The first method involves propelling the ball through air and measuring the trajectory of the ball. This may involve the ball being propelled through 'still' air using a high-precision cannon [7], or being dropped into the working section of a wind tunnel [8]. The main problem associated with this technique is that the sampling of the ball displacement is susceptible to a finite error. The calculated force which acts on the ball is extremely sensitive to the accuracy of the sampled ball displacement data. This aspect of this experiment means that large errors in the force calculation are observed which increase the uncertainty of the final results. Furthermore, in this experiment, it must be assumed that, for example, the spin remains constant throughout the test.

An alternative method involves supporting the ball on a rotating force balance inside the wind tunnel test section. This enables the ball to be spun in a flow of air, and the resultant force which acts on it can be sampled directly. The main consideration for this type of experiment, is to ensure that the ball supports do not interfere with the flow of air over the ball.

Various authors [9,10] have supported a rotating sphere in a wind tunnel. These two papers describe two different methods which can be used to spin the sphere. Bearman and Harvey supported an oversize golf ball (100 mm diameter) at its centre by a vertical, taut wire (0.5 mm diameter) and the model was spun by a motor assembly housed inside the model. The thin wire minimises interference on the flow of air over the ball. Watts and Ferrer supported a baseball (74 mm diameter) using two rigid, horizontal stings (6.3 mm diameter). Bearman and Harvey measured the C_D of a smooth sphere using their apparatus, and found that it was equal to approximately 0.51 thus proving that the supports did not interfere with the flow of air over the ball. However, Watts and Ferrer did not perform this experiment, and therefore it is not possible to conclude whether the data quoted in their paper was influenced by the presence of the stings.

The intricate spinning method used by Bearman and Harvey would be unsuitable for this study as it would be impossible to place the motor inside the tennis ball, without damaging the felt in some way. However, a method similar to that used by Watts and Ferrer would be suitable, and this method is described in the following section.

2.1. *Wind tunnel details*

The wind tunnel was supplied by Aerotech ATE Ltd. A slotted wall test section has been fitted to reduce blockage effects normally associated with closed wall test sections. The test section (305 mm $\times$ 355 mm) is fitted with 30% open area ratio walls which is based upon the technology developed for automotive wind tunnels [11,12]. A honeycomb section settling chamber with a contraction ratio of 10:1 and two screens, results in high-quality flow.

The velocity variation along the entire test section was $\pm 0.25\%$, across the entire test section was $\pm 0.3\%$ and the turbulence intensity in the test section was 0.3%. The wind tunnel was operated over a range of wind speeds between 20 and 60 $\mathrm{m s^{-1}}$ (45 and 135 mph), which correspond to Re of between 80×10^3 and 250×10^3. Although the highest recorded service speed is 149 mph [13,14] showed that the average first serve speeds are closer to 110 mph for top male professionals. Stepanek [4] asked two Davis Cup players to hit a 'heavy' top-spin shot and measured maximum spin rates of 3000 rpm. These two studies illustrate typical speeds and spins which should be used in this study.

The ball is supported in the centre of the test section using two horizontal stings, as shown in Fig. 1. The stings have a stepped profile to reduce the interference around the ball whilst maximising the strength of the sting and minimising the vibration of the ball. Each sting has a diameter of 3.5 mm and extends up to 35 mm away from each side of the ball. Preliminary testing showed that this is the critical area in which the interference of the air flow over the ball must be minimised. A further 85 mm of the sting is shrouded by a elliptical shroud with a height and width

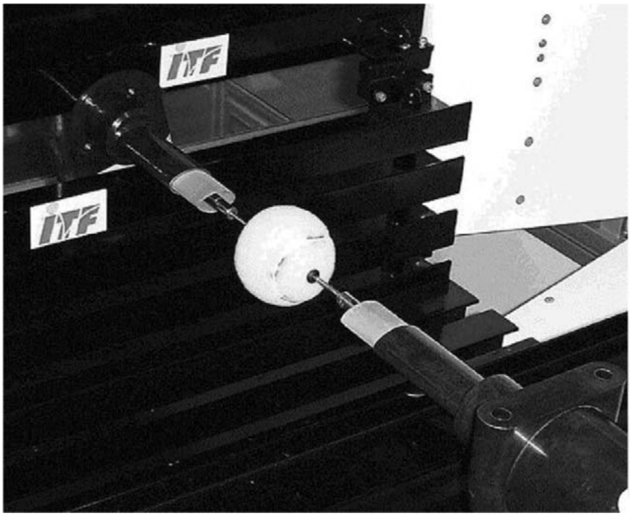


Fig. 1. Ball supported in working section of the wind tunnel.

of 20 and 35 mm, respectively, to reduce further the load which could act on the sting. This resulted in a test section blockage of only about 6% (based on cross-sectional area) which is negligible for a 30% open area slotted working section [12], and therefore blockage corrections were not applied.

The support mechanism shown in Fig. 1 allowed the ball to be rotated at a constant angular velocity whilst simultaneously measuring the drag and lift forces. The motor and bearing housings both had to be supported by the balances to ensure that the load was correctly transferred to the balance. Each balance was housed in a sealed unit and was mounted on vertical struts which were located outside the slotted wall test section. The balances were bespoke units supplied by Aerotech ATE Ltd. Each channel of the two component force balance was sampled and routed into dedicated sensor units which provide digital, high-resolution output from each channel. Communication to the NI FlexMotion (PCI-6C) data acquisition system was via RS485 serial communication ports and bespoke software was used to read the signals. The force balance had been previously calibrated by Aerotech and was checked prior to the commencement of each test. The magnitude of the uncertainty in the measurement of load was of the order of 10^{-4} N, and therefore insignificant in the calculation of C_D.

The poles of a tennis ball were marked using a purpose built rig. A hole was drilled at one pole and the ball was then filled with a polyurethane foam to ensure structural stability of the balls during testing. Another hole was then drilled into the other pole of the ball. Small plastic inserts were secured in the holes using integral cross-headed screws. The ends of the stings were fitted with posi-drive screw driver ends which sat in the cross-headed screws and enabled the torque from the motors to be transferred to the ball. One sting was spring loaded which allowed easy installation of the ball.

The diameter of the ball was determined using Vernier callipers, following Mehta and Pallis [6]. The ball was gripped between the callipers and the callipers were slowly released. The measurement was recorded when the ball dropped, under its own weight, through the callipers. An average diameter was determined from measurements across several axes through the ball. This averaging technique was adopted due to the non-homogeneity of the ball, and to minimise the errors associated with the use of the Vernier callipers in measuring a non-rigid object. The repeatability of the measurement was ± 0.2 mm.

2.2. *Testing procedure*

In all the experiments, the air stream temperature and pressure were measured using high-accuracy transducers (Type LM35DZ and SensorTechnics PTE5001D, respectively), which were sampled by the PC control unit. The wind speed was calculated using the differential pressure measured between two pressure tapings, which were located in the settling chamber and the plenum. This differential pressure was regularly compared with the total and static pressures measured using a pitot-static tube placed at the centre of the empty test section. The maximum uncertainty in the wind speed measurement was ± 0.1 m/s at 20 m/s. When the wind tunnel had settled to a constant speed, the wind speed and force balance signal were sampled at 10 counts per second, for 10 s. Mean values of these two parameters were calculated to minimise the noise in the data.

2.2.1. *Non-spinning balls*

The loading due to the stings is accounted for in the calculation of C_D by the tare loading of the system. The tare ball is placed in the position of the test ball, supported on a separate vertical sting, to ensure that the effect on the flow of air due to the presence of the ball is accounted for. This ball has holes drilled at the poles, where the inserts would normally be placed. It is important to ensure that the tare ball does not touch either of the horizontal stings, so that the load measured by the force balances is solely due to that of the drag on the stings.

2.2.2. *Spinning balls*

Chadwick [15] reported that the tare drag which acts on the stings is not a function of the spin rate of the stings. Therefore, the tare drag data collected for the non-spinning ball was also used for the spinning ball experiments.

In the spinning ball tests, the calculation of C_L needs to account for the tare lift force which acts on the spinning stings. This force is *not* obtained easily due to the safety features incorporated into the control software of the ITF wind tunnel. These safety features prevented the stings from spinning when the ball was not in place. However, the magnitude of these loads can be estimated using the data published by Chadwick [15], in which a very similar experiment was conducted. Chadwick estimated that the maximum lift forces acting on the stings are only 0.016 and 0.033 N at wind speeds of 25 and 50 m/s, respectively, at the maximum spin rate of 3000 rpm.

Preliminary work by the authors revealed that the tare loads calculated using the data of Chadwick represent between 4% and 6% of the lift load measured by the force balances. Therefore, as an approximation, the tare lift load can be assumed to be equal to 5% of the sampled lift load from the force balance.

The lift and drag loads which acted on the balance when the shafts were spun at zero wind speed were obtained for the full range of spin rates. These loads increased with spin rate, but the maximum value was less than 0.05 N. These loads were accounted for in the analysis of the measurements of C_D and C_L of spinning tennis balls.

2.3. Flow interference

In order to verify the accuracy of the force balance and wind speed measurement system, an additional test was conducted using a smooth epoxy-resin sphere with a similar diameter ($d = 66.0$ mm) as a standard tennis ball. The results in Fig. 2 compare reasonably well with the classic data of Achenbach [17], who reported a constant C_D of 0.51 ± 0.01 over the range of Re tested here. In this study, the smooth sphere exhibits a constant C_D of approximately 0.54 ± 0.01, which is about 5% higher than that reported by Achenbach. This difference may be associated with errors in the measurement of wind speed or drag load. Alternatively, it may be caused by interference of the flow pattern over the ball, due to the presence of the support stings. Achenbach's data was obtained using a sphere that was mounted from the rear to minimise interference. To recreate Achenbach's experiment in the Aerotech ATE wind tunnel, the Aerotech force balance (Fig. 1) was removed and replaced with a three-component balance manufactured by TEM Engineering Ltd. This three-component balance supported the smooth sphere from the rear. Using this arrangement, a constant C_D of 0.51 ± 0.01 over the range of Re tested here, was obtained. This result implied that the higher than expected values of C_D for a smooth

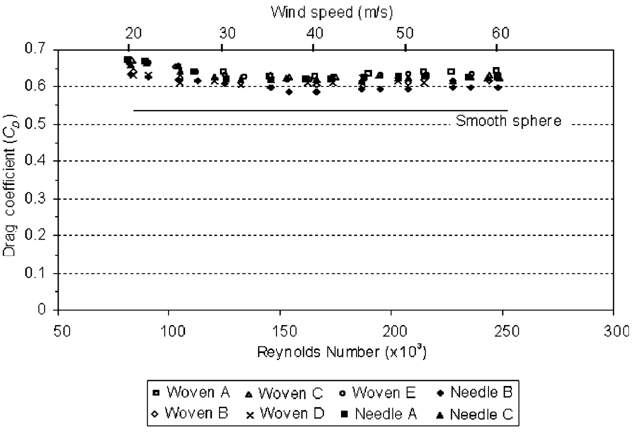


Fig. 2. Drag coefficient of eight different brands of new tennis ball.

sphere, as shown in Fig. 2, were due to flow interference caused by the presence of the side supports.

It has been shown above that the side supports (in Fig. 1) affect the flow over the sphere/ball, as indicated by the C_D values which were found to be 5% higher than expected. This is an inherent problem associated with this method of supporting a spinning object in a wind tunnel. Several other designs of side support were tried by the authors, and this was the optimum configuration. It is assumed that this will be a systematic error that affects all ball types equally. Therefore, the results obtained will still provide a valid comparison of the aerodynamic properties of various types of tennis ball.

2.4. Balls tested

A large selection of tennis balls were tested in this study to cover the wide range of balls used in the game of tennis. Tennis balls are typically covered with a felt which defines them either as woven cloth or needle cloth balls. Woven cloths are produced by the long-established processes of weaving, raising and fulling [16]. These cloths have a high wool content (50–65%) and the remainder of the fibre content is made up of nylon. Woven cloths are used predominantly for top quality balls. Needle cloths are produced with a mat of randomly orientated fibres which are progressively pushed into a base cloth by a large number of barbed needles which cyclically travel through the fibres. This action gradually produces a continuous interlocking structure held together by inter-fibre friction. Needle cloths are generally cheaper to produce when compared with woven cloth. However, they tend to lose their nap more quickly and are mainly used at the lower end of the tennis ball market.

In this study, five different ball brands which are covered in woven cloth (Melton) and three ball brands which are covered in a needle cloth were tested. A tube (3 balls) of each of the eight brands was purchased off the shelf for the purpose of this testing.

A selection of worn tennis balls were also tested in this study. There is no fixed amount of impacts to which a ball is subjected to during a point, game or match. However, if it is assumed that on average there are four shots per point and six points per game, and six balls are used at a time, then each ball is only actually subjected to four shots per game. In the professional game, the balls are changed every nine games, hence each ball, on average, is subjected to just 36 shots. In preliminary testing by the authors, it was found that C_D was unaffected by the number of impacts that the ball was subjected to, for impacts up to 60 shots. This contradicts the findings of Mehta and Pallis [6] who showed that the C_D of a tennis ball changed significantly with the number of impacts that a ball was subjected to. This may be due to the different way in which wear was achieved in both studies.

The aim of the wear testing conducted in the current study was to investigate whether the aerodynamic properties of a tennis ball would change significantly with prolonged use. This would be of most interest to a leisure standard player who is likely to use the same ball for a far longer period than a professional player. However, it would also help fulfil the aim of this study which was to benchmark the aerodynamic properties of a tennis ball.

In this study, the wear of a tennis ball was simulated using the 'wear rig' at the ITF laboratory. The ball is projected from an air cannon at a velocity of 30 m/s onto an acrylic surface at an angle of 16°. The ball then impacted on to a rigid surface which directed it back into the cannon hopper. The process was repeated for the specified number of impacts. This rig is assumed to replicate a typical tennis shot where the ball hits the surface, and is then hit by the racket. It was arbitrarily chosen for the balls to be worn for 60, 500, 1000 and 1500 impacts, to correspond to between 2 and 50 games if only one ball is used in the game. The diameter of the balls was measured before and after wear testing. It was expected that wear might have reduced the measured diameter due to loss of fibres but this was not noticed, as any differences were smaller than the repeatability of the measurement.

2.5. Theory

The aerodynamic forces acting on a spinning sphere are functions of both the sphere characteristics and the fluid which surrounds it and therefore

$$F_\mathrm{D},\ F_\mathrm{L} = f(d,\ \rho,\ \mu,\ U,\ V) \tag{1}$$

and from the rules of dimensional analysis

$$C_\mathrm{D},\ C_\mathrm{L} = f[\mathrm{Re},\ V/U]. \tag{2}$$

Eq. (2) shows that both C_D and C_L are functions of *Re* and the spin coefficient S ($= V/U$) for a spinning sphere. For a non-spinning sphere, C_D and C_L are functions of *Re* only.

2.6. Repeatability of C_D measurements

In this study, three balls of each ball type were used and C_D values were obtained for two separate runs for each ball. For each run, the balls were tested for a wind speed range between 20 and 60 m/s, at increments of approximately 2 m/s. The scatter in the measurement at each increment was ± 0.01, for the two runs. However, when second-order polynomial trendlines were plotted through the data for each run, the maximum difference between the two trendlines was less than 0.005. This illustrates that despite the random errors in the individual measurements, there is negligible bias in the results for each run.

The repeatability of C_D for the three balls of each type was typically ± 0.02. Mehta and Pallis [6] observed a similar variation and attributed some of the difference to the unsteady nature of the flow field, due to vortex shedding in the wake. They also investigated the effects of seam orientation but found it to have a negligible effect on C_D. Although high-precision instruments have been used to measure C_D of tennis balls, the consistency of the results for three balls of the same type will be affected by the fact that the ball is not a rigid object. Chadwick and Haake [5] showed that the fluff on a ball is a major contributor to the drag coefficient. Therefore it can be postulated that some of the scatter which was observed between balls of each type may simply be due to the random orientation of the fluff caused by differences in the handling and storage of the balls.

To summarise, it was found that the results for a specific ball repeated very well, despite some random error in individual measurements. However, the repeatability of C_D between balls of the same type was typically ± 0.02.

3. Results—non-spinning balls

Fig. 2 shows the drag coefficient values for all the new tennis balls and a smooth sphere. The wind speed quoted on the upper abscissa is that for a standard sized tennis ball ($d = 65.0$ mm). For clarity, the average C_D values for the three balls of each type have been used. It can be seen that C_D of a tennis ball reduces from a value of approximately 0.66 at Re of 80×10^3, to a value of approximately 0.62 for $125 \times 10^3 < Re < 250 \times 10^3$. This is in agreement with those found by Mehta and Pallis [6] over a similar range of Re. The reduction in C_D as Re increases is a well established result and can simply be assigned to the nap 'laying down' at the higher wind speeds. A reduction in C_D of 0.04 would be measured if the diameter of the projected area of the ball reduced by 2.0 mm. Images of the nap 'laying down' at high wind speeds [15] confirmed that this reduction in diameter was of the correct order of magnitude.

Fig. 2 shows that there is no discernable difference in C_D of all the different ball brands, except the Woven B and Needle B balls. The C_D of these two balls is typically 0.01–0.02 lower than that of the other ball brands, which is within the repeatability of the data. Hence, it can *not* be concluded that there is any significant difference between ball brands, a conclusion also reached by Mehta and Pallis [6].

Fig. 3 shows the drag coefficient data for the worn tennis balls. Two balls were tested for each number of impacts and the average C_D are plotted in this figure. It can be seen that the drag coefficient again reduces with Re. This has previously been associated with the fluff 'lying down', for the new balls. There was little fluff left on the heavily worn ball (1500 impacts), but the majority of the base felt remained. This

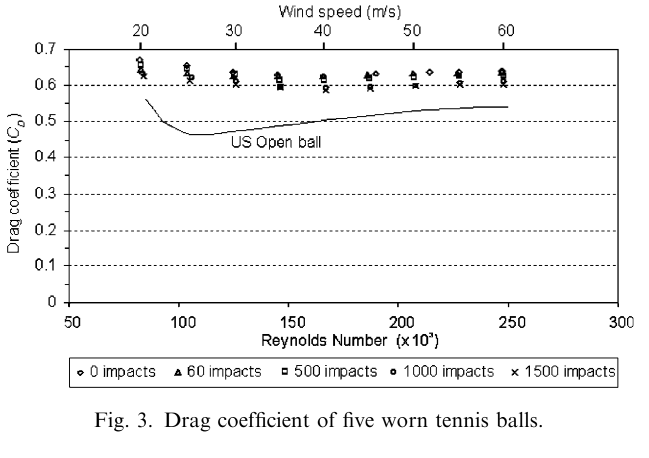


Fig. 3. Drag coefficient of five worn tennis balls.

fabric surface has a very low stiffness and is easily deformable. The reduction in C_D with *Re*, for this worn ball, may be due to this surface deforming which would result in a decrease in the projected area of the ball. Alternatively, the reduction in C_D with *Re* may not be due to a physical change in the diameter of the ball, but may be due to a change in the surface roughness of the ball. Mehta and Pallis [6] explained that the flow over a tennis ball is in the transcritical regime, where C_D is essentially independent of *Re*. Achenbach [18] showed that, in the transcritical regime, C_D for a rough sphere decreases as the surface roughness reduces. It could be postulated that the wind flowing over a tennis ball acts to reduce the surface roughness, by pushing any loose fibres onto the base felt. As the wind speed is increased, the surface roughness will decrease, and this would be accompanied by a drop in C_D.

Fig. 3 shows similar trends for all the number of impacts, suggesting that the relationship between C_D and the number of impacts is independent of *Re*. A clearer indication of this is given in Fig. 4, in which C_D is plotted as a function of the number of impacts, for two different wind speeds. It can be seen that C_D remains approximately constant for balls subjected to 0, 60 and 500 impacts. The drag coefficient of the ball subjected to 1500 impacts is 0.04 lower than that of the new ball.

The data presented by Mehta and Pallis [6] for a ball used in the US Open is also shown in Fig. 3. It can be seen that C_D of this ball is significantly lower than that of all the balls which were tested in this study. The C_D of the US Open ball is typically $\leqslant 0.1$ lower than that of the new ball tested in this study. Even the most heavily worn ball (1500 impacts) only exhibited a drop of 0.04 in C_D in this study. Without observing the surface of the US Open ball, it is difficult to hypothesis why they obtained very low values of C_D.

An interesting observation from Figs. 2 and 3 is that, for the range where *Re* increases from 175×10^3 to 250×10^3, C_D actually rises by approximately 0.01. This

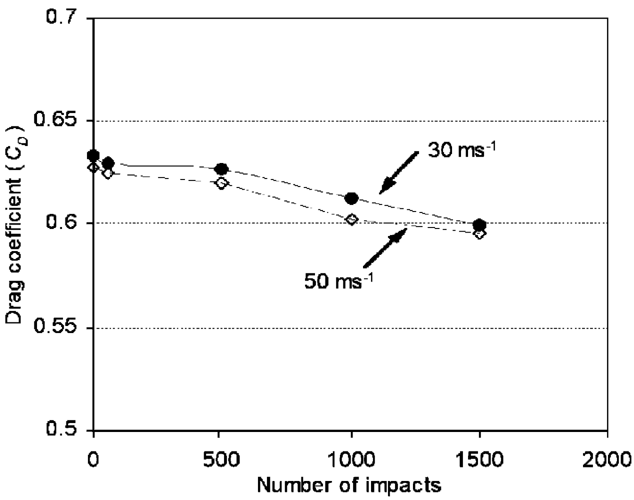


Fig. 4. Effect of wear on C_D for two different wind speeds.

rise in C_D is only very small and therefore may not be significant. However, this finding was also evident in the work by Mehta and Pallis [6] and Chadwick and Haake [8] but no comment was made in either of these studies.

3.1. Implication to the game of tennis

The wind tunnel has been successfully used here to determine C_D for non-spinning tennis balls. This data, although valuable in its own right, does not quantify the differences that a player will notice when using the balls. Accurate trajectory plots are a far more useful method of illustrating the differences in C_D, as shown by Haake et al. [19]. The authors have developed the *TennisGUT* modelling software to model the mechanics of the game of tennis including the impact between the ball and racket, and that between the ball and surface. The trajectory of the ball is governed by simple Newtonian mechanics, with inputs from experimentally obtained values of C_D and C_L.

To illustrate the effect of C_D on the game of a tennis, a typical flat first serve (non-spinning) was simulated. It was assumed that the ball left the racket at a speed of 120 mph (50.0 m/s), 6.2° below the horizontal and 2.7 m above the ground, the ball rebounded from the surface with a coefficient of restitution of 0.7, and a rebound angle ratio of unity. These parameters simulated an impact on an acrylic surface [19].

Fig. 3 shows that C_D remains approximately independent of *Re* for wind speeds above 30 ms^{-1}, and therefore it was assumed that C_D was constant throughout the trajectory of the serve being simulated here. Hence the C_D of a new (0 impacts) and a heavily worn (1500 impacts) ball tested here are 0.64 and 0.6, respectively, for the serve being simulated. For comparison, the US Open ball used by Mehta and Pallis [20] has a constant C_D equal to 0.54 for the simulated serve. The trajectories of these three balls are plotted in Fig. 5. Each of these three standard size balls was assumed to have a diameter of 65.0 mm, and a mass of 0.057 kg. For comparison, the trajectory of an oversize ball (6.5% larger diameter and identical mass) is also shown. Chadwick and Haake [5] and Mehta and Pallis [6] concluded that an oversize ball has the same C_D as a standard ball. Therefore, the oversize ball in Fig. 5 has a C_D of 0.64. In the trajectory plots, any differences in flight are due to either C_D or the ball diameter.

The trajectory was terminated after 0.68 s which is approximately the time taken for the 'fastest' ball to reach the baseline. It can be seen that the ball subjected to 1500 impacts ($C_D = 0.60$) travels approximately 300 mm further than the new ball ($C_D = 0.64$), the US Open ($C_D = 0.54$) travels 750 mm further than the new ball, and the oversize ball travels 600 mm shorter than the new ball.

In a similar analysis, Haake et al. [19] concluded that the player would perceive a significant increase in the reaction time for an oversize ball, when compared to a standard ball (with the same C_D). This is supported by anecdotal evidence in which players claim that the larger ball is slower, and gives them more time to play their shot.

Using the trajectory plots in Fig. 5, this would imply that a player would have a significantly shorter reaction time if the US Open ball was used in a game, compared

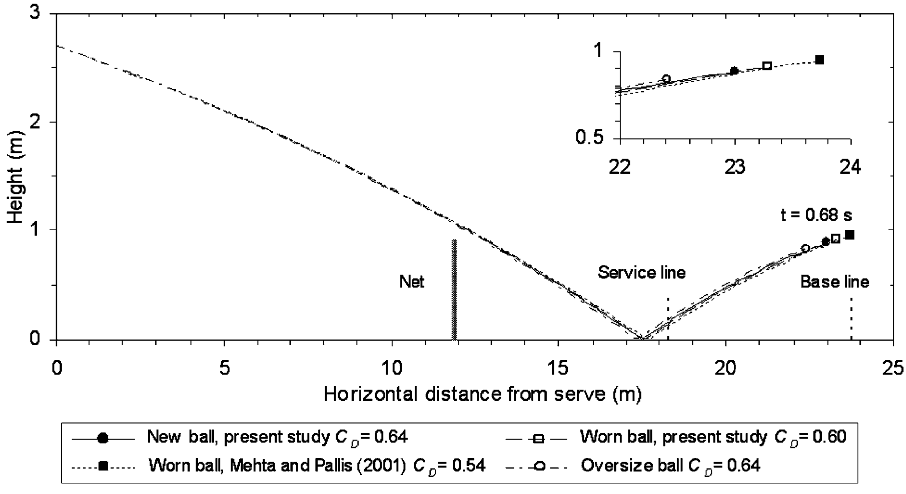


Fig. 5. Predicted trajectory for three standard size tennis balls with different C_D values, and a new oversize tennis ball with the same C_D as a new standard ball. The inset shows a close-up view of the end of the trajectory ($t = 0.68$ s).

with a new ball. Furthermore, the ball subjected to 1500 impacts would reduce the receiver's reaction time but only by approximately 40% of that experienced when using the US Open ball.

The trajectory plots in Fig. 5 illustrate the effect that C_D and ball size have on the game of tennis, for a typical first serve. The motivation for the development of an oversize tennis ball was the concern that the serving speed in the men's professional game has increased to a point where the serve dominates the game. More specifically, the particular concern was that on fast surfaces, such as the grass courts at Wimbledon, the game would become boring with receivers unable to return a high proportion of the first serves. Haake et al. [19] illustrated that an oversize ball would give a receiver a significantly longer time to play their shot. Conversely, the analysis in this study has shown that a heavily worn tennis ball can actually magnify the dominance of the server.

It has been found that C_D changes with wear but diameter remains unchanged. This implies that a manufacturer could make a ball which complies with the ITF ruling on size, but exhibits lower drag forces. This ball would still 'look like a tennis ball' and manufacturers could claim that it 'travels through the air faster'.

4. Results—spinning tennis balls

4.1. Drag coefficient

In this section, the drag and lift coefficients of the balls are presented for two different Re (105×10^3 and 210×10^3). For each Re, the spin was increased from 0 to 2750 rpm, in 250 rpm increment. Two runs were conducted for each ball, and three balls of each type were tested. The repeatability of C_D for each run was typically ± 0.015, and between the three balls of each type was typically ± 0.025. This repeatability was worse than that for the non-spinning ball tests, and this was likely to be due to the unsteady flow field caused by the vibration of the spinning ball. The vibration of the ball was minimised by filling the core with a rigid foam and by aligning the poles using an accurate jig. However, a tennis ball is a non-homogenous object and a finite out-of-balance force will be inherent when the ball is spun.

Figs. 6a and b show that the relationship between C_D and the spin coefficient for Re of 105×10^3 and 210×10^3, respectively. The spin coefficient S $(= V/U)$ is smaller in Fig. 6b, compared to Fig. 6a, due to the higher U being used in this test. Both figures show that the maximum difference between the different balls for the range of S tested is within the repeatability of the measurement. Therefore, it can only be concluded that all the balls behave similarly for the range of S tested.

The values of C_D at zero spin ($S = 0$) corresponded to within approximately ± 0.02 of those obtained in the non-spinning tests, thus verifying that both sets of data are consistent. It can also be seen that the drag coefficient for each ball type is higher in Fig. 6a compared to that in Fig. 6b, for the same spin coefficient S. This confirms that C_D is not solely a function of S, but is also dependent on Re.

Both Figs. 6a and b show that C_D increases with the spin coefficient. Fig. 6a shows a significant increase from 0.65 to 0.69 for the range of spin coefficients. The increase in C_D with spin rate is partly attributed to the induced drag force that is due to the generation of a lift (Magnus) force. Also, the surface of a tennis ball is not a rigid object, and when caused to rotate, the fluff on the surface will attempt to 'stand up', thus increasing the projected area of the ball in the wind flow direction. This will inherently increase the drag force, and thus the calculated drag coefficient, when the spin coefficient is increased.

The previous section shows that C_D for a non-spinning ball decreases by approximately 0.04, for balls worn by 1500 impacts, compared to new tennis balls. Spinning balls would be expected to show a similar difference in C_D with number of impacts. This testing was conducted at two different wind speeds and the results are presented in Figs. 7a and b.

In Fig. 7a it can be seen that C_D is dependent on the number of impacts that a ball is subjected to. The balls subjected to 0 and 60 impacts act very similarly to all the new balls in Fig. 6a, and there is no significant difference between these two balls. The drag coefficient of the balls subjected to 0 and 60 impacts increases with S due to the increased projected area of the ball when it spins and the fluff begins to 'stand up'. Chadwick [15] confirmed that the cloth on a new ball 'stands up' when it is spun in a flow of air. The balls subjected to 500 and 1000 impacts exhibit a slight rise in C_D with S, but to a much smaller extent when compared to the new balls. The largest difference between the balls can be seen for a spin coefficient $S = 0.15$. At this value of S, the C_D of the heavily worn ball (1500 impacts) is approximately 0.61 whereas the C_D of the new ball is 0.67. The C_D of the worn balls (500, 1000 and 1500 impacts) decreases in the range where S is increased from 0 to 0.2. It can be concluded that the C_D of the new balls is at least 0.04 higher than that of the heavily worn balls (1500 impacts).

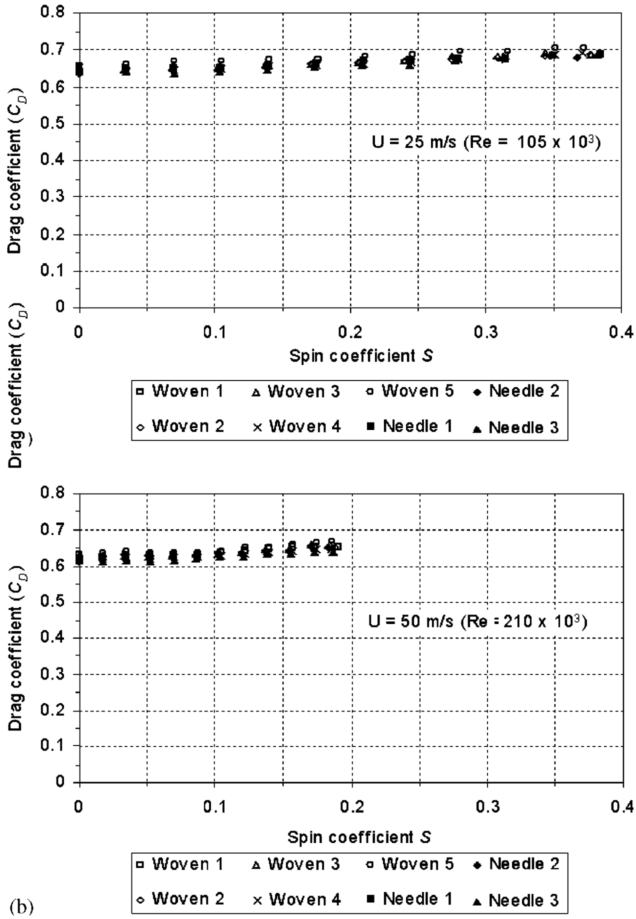


Fig. 6. Drag coefficient for the eight different new tennis balls. The data was obtained at wind speeds of (a) 25 m/s ($Re = 105 \times 10^3$) and (b) 50 m/s ($Re = 210 \times 10^3$).

Fig. 7b shows the drag coefficient for the wind speed of 50 ms^{-1}. The drag coefficient tends to increase slightly with increasing S, for all the worn balls. However, it should be noted that this effect is small for the range of S presented in Fig. 7b. It can be seen that C_D for the new ball is 0.03–0.04 higher than that of the heavily worn ball, for all values of S. This difference is of a similar order of magnitude as that found for non-spinning balls.

4.2. *Lift coefficient*

A sphere spinning in a flow of air will exhibit both drag and lift forces. The lift force can be explained by the conventional Magnus effect because previous research has shown that a turbulent boundary layer separation would be expected for all *Re*

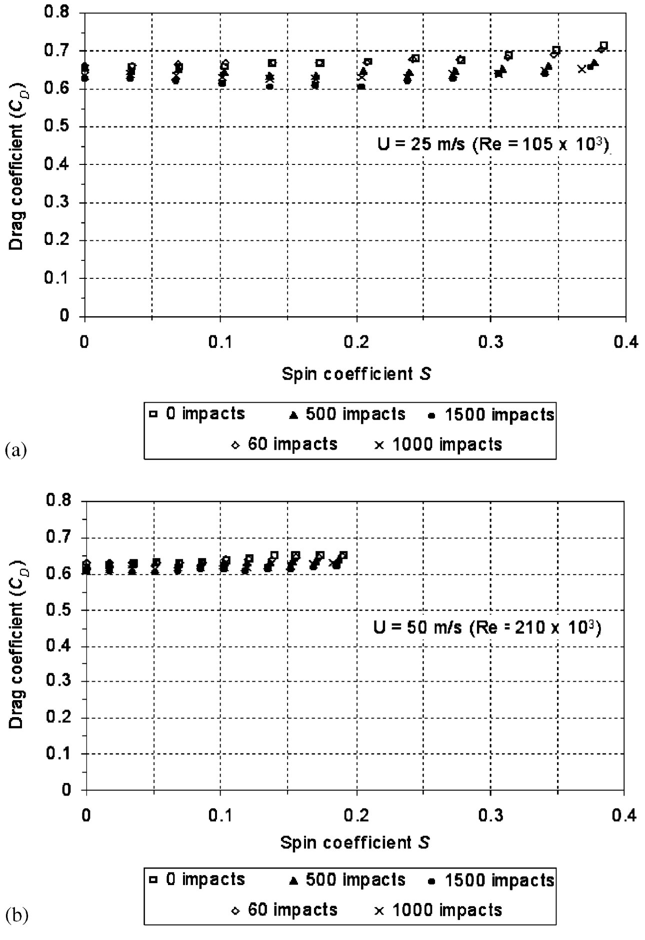


Fig. 7. Drag coefficient for the five worn tennis balls. The data was obtained at (a) $U = 25\,\mathrm{m\,s^{-1}}$ ($Re = 105 \times 10^3$) and (b) $U = 50\,\mathrm{m\,s^{-1}}$ ($Re = 210 \times 10^3$).

values tested in this study [6]. When the sphere spins, the separation point moves upstream on the advancing side of the ball, and downstream on the retreating side of the ball causing an asymmetric pressure distribution. This asymmetry causes a resultant force acting in the direction of the advancing side of the ball [15]. This force would cause a ball hit with top-spin to dip in flight. The faster the ball spins (relative to its speed), the greater the force and the lift coefficient C_L increases.

The lift coefficient C_L for the eight new tennis balls is given in Fig. 8a and b, and is presented separately for the two different wind speeds. Fig. 8a illustrates the data for the lower wind speed. It can be seen that there is no noticeable difference in the values, for all the different ball types. Also plotted in Fig. 8a is the lift coefficient data obtained by Chadwick [15] which is significantly lower than that obtained in this current study. Chadwick obtained a single relationship between C_L and S, for wind speeds up to $26\,\mathrm{ms}^{-1}$. However, it should be noted that Chadwick used a 'modified' ball diameter of 73 mm in the calculation of C_L, whereas the typical diameter used in this study is 65 mm. Chadwick used this larger diameter in an attempt to simulate the increase in projected area when a ball is spun and the fluff stands up. A simple calculation can be performed to illustrate that if Chadwick had used a ball diameter of 65 mm instead of 73 mm, then Chadwick's C_L values which are shown in Fig. 8 would be increased by 26%. This would make Chadwick's data identical to that measured here.

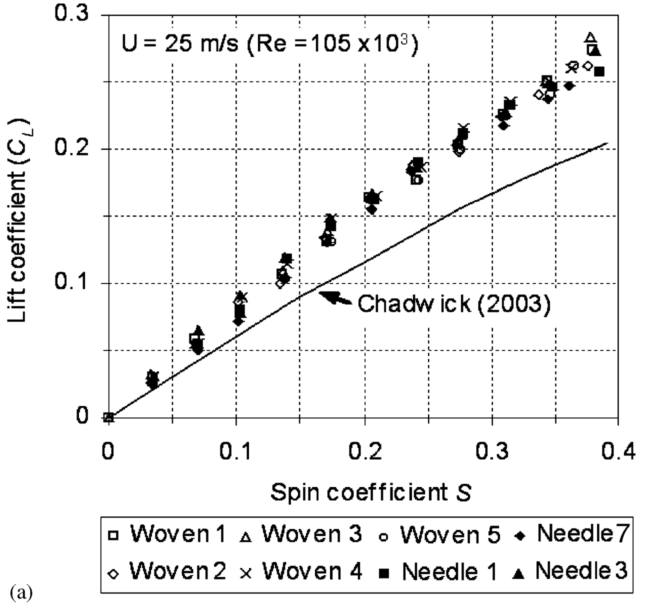
Fig. 8b illustrates the values of C_L which were measured at the higher Re. It can be seen again that there is no noticeable difference in the data for all ball types. Comparing Figs. 8a and b it can be seen that, for $S<0.1$, the measured values of C_L are lower at the higher wind speed compared to the lower wind speed. The implies that C_L is dependent on Re, for $S<0.1$. However, for $0.1<S<0.2$, it can be seen that C_L is similar in both figures, which implies that C_L is independent of Re.

The lift coefficient C_L for the five worn tennis balls, measured at a wind speed of $25\,\mathrm{ms}^{-1}$, is given in Fig. 9a. It can be seen that, for $0.05<S<0.15$, the value of C_L is significantly lower for the balls which have been subjected to 500, 1000 and 1500 impacts, compared to the new balls. For $S>0.15$, the lift coefficient is approximately constant for all the balls.

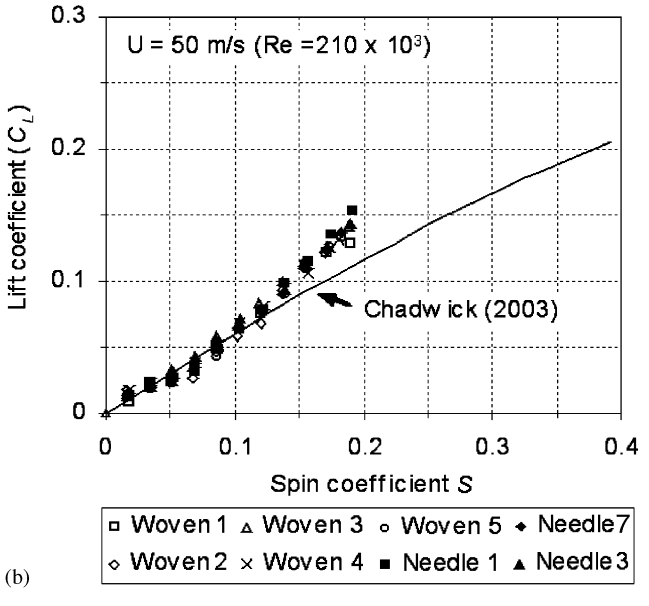
Fig. 9b, which shows the data for the higher wind speed, does not show the same trends as those which are evident in Fig. 9a. In this figure, for $S>0.1$, the C_L is highest for the new ball (0 impacts), and lowest for the heavily worn ball (1500 impacts). The maximum difference between these two balls is 0.03, which is similar to the repeatability of the data.

To summarise, it has been found that C_D and C_L for a worn tennis ball are generally lower than or equal to that of a new ball. This difference must be attributable to the surface of the balls. Both balls have the same diameter, but simple observation of the balls reveals that the new balls have more fluff. Mehta and Pallis [6] concluded that this fluff has a significant contribution to the total drag on the ball by effectively increasing the size of the wake. The lift force, on a spinning ball, is generated from the deflection of this wake, and so the magnitude of C_L will be dependent on the size of this wake. If it is assumed that the separation points move through an equivalent angle on both the new and used balls, then clearly the new ball will be subjected to a larger lift coefficient.

It can be seen that the differences between the new and worn balls is smaller at the higher wind speed, for both C_D and C_L. This is not surprising because, at this higher speed, the fluff on the new balls will be forced to lay down by the air flow. Therefore the surfaces of the new and worn balls will be similar and the differences between the balls will be minimal. Furthermore, comparing Figs. 9a and b, it can be seen that a worn ball tested at $S=0.1$ and $U=25\,\mathrm{ms}^{-1}$, acts very similarly to a new ball tested at $S=0.1$ and $U=50\,\mathrm{ms}^{-1}$. This implies that the structure of the surface will be similar for the two balls, i.e. a new ball tested at a high wind speed, has surface characteristics which are similar to a worn ball tested at low speed.



(a)



(b)

Fig. 8. Lift coefficient for the eight different new tennis balls. The data was obtained at (a) $U = 25\,\mathrm{ms}^{-1}$ ($Re = 105 \times 10^3$) and (b) $U = 50\,\mathrm{ms}^{-1}$ ($Re = 210 \times 10^3$).

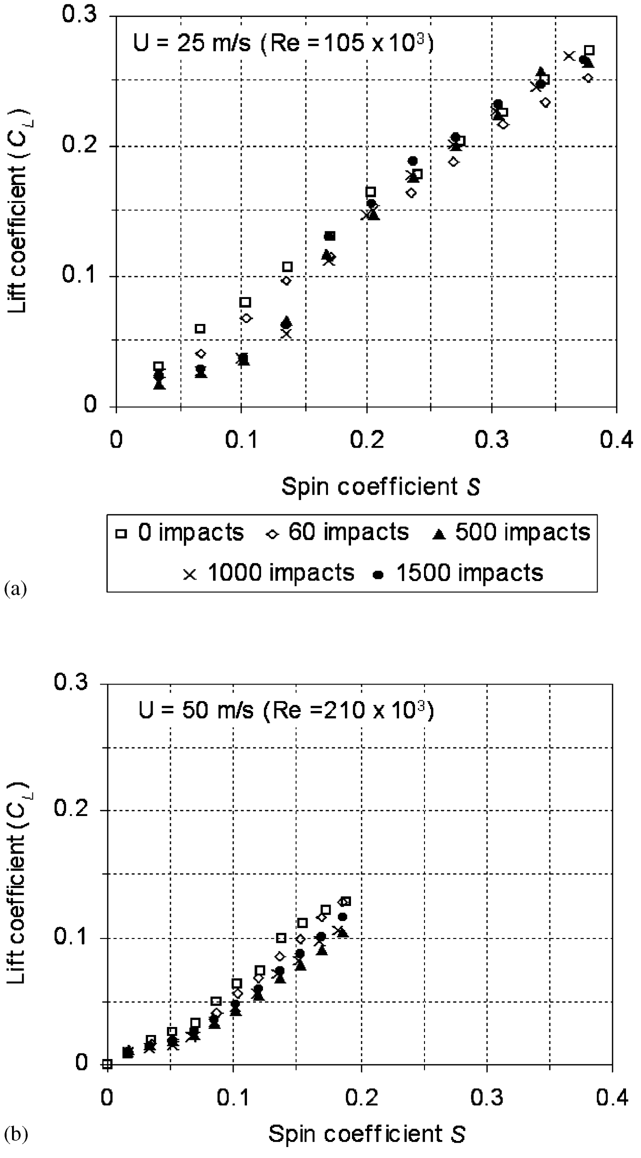


Fig. 9. Lift coefficient for five worn tennis balls. The data was obtained at (a) $U = 25\,\mathrm{ms}^{-1}$ ($Re = 105 \times 10^3$) and (b) $U = 50\,\mathrm{ms}^{-1}$ ($Re = 210 \times 10^3$).

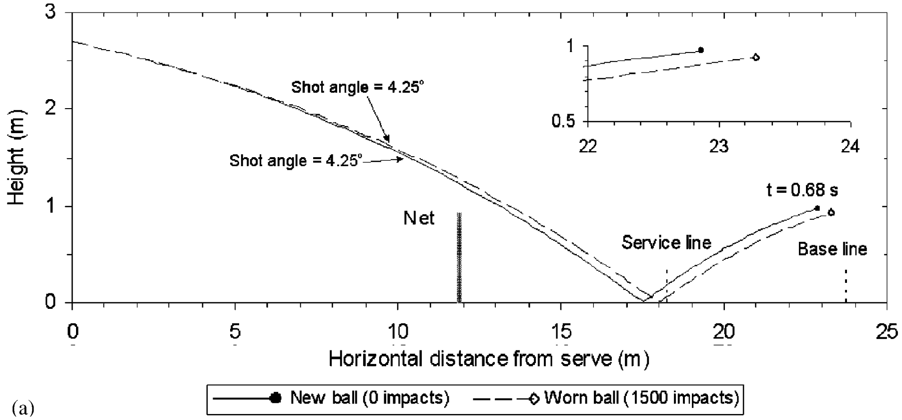
It has been found that C_D and C_L are dependent on Re, as well as the spin coefficient. This would be expected because surface condition will change with wind speed (Re). This finding appears to contradict the conclusions of Stepanek [4] and Chadwick [15] who used different apparatus to that used in this study. The conclusions in [4,5] would have been useful as it meant a considerable reduction in experimentation required to quantify C_D and C_L. Furthermore, it can be concluded that the testing needs to be conducted over a wide range of *Re*, to be able to categorise the aerodynamic properties of a spinning tennis ball.

4.3. Implication to the game of tennis

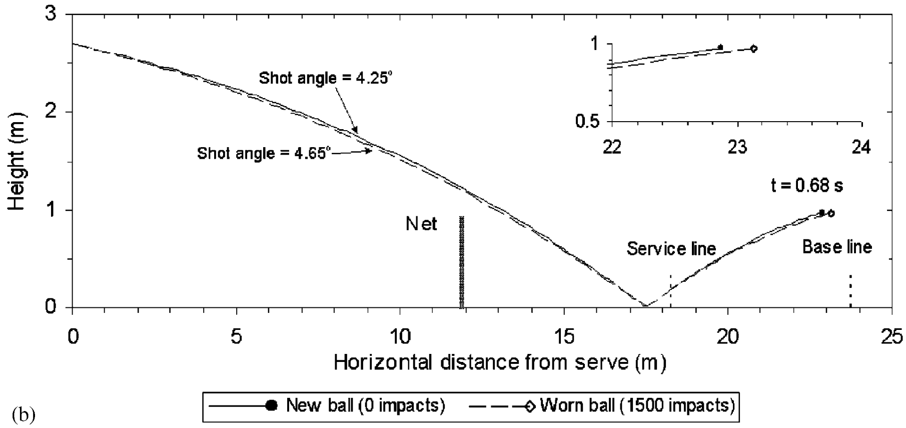
It has been shown that all new tennis balls exhibit similar values of C_D and C_L, when spun in a wind tunnel, and that they are different for new (0 impacts) and heavily worn (1500 impacts). To illustrate the effects of the differences in C_D and C_L for a new and heavily worn tennis ball, trajectories of the two balls have been calculated for a typical first serve, hit with top spin. It was assumed that the ball left the racket at a speed of 113 mph (50.0 m/s), 4.25° below the horizontal and 2.7 m above the ground. The initial spin on the ball was 2000 rpm which gives a spin coefficient of 0.11. The values of C_D and C_L, for these initial conditions, can easily be determined from Figs. 7b and 9b. This shot is similar to the flat serve that was simulated in the previous section. Using that trajectory model it can be shown that the ball has slowed down to 21 ms^{-1} when it arrives at the baseline. This illustrates that the ball speed (*Re*) varies widely during the trajectory, and it has been shown that the values of C_D and C_L vary with *Re*. Clearly the variation in the ball speed during the trajectory makes it invalid to use the data collected at 50 m/s, for the full trajectory. In order for the trajectory of the serve to be calculated in this study, a linear interpolation between the results for a wind speed of 25 and 50 ms^{-1} was assumed, for $S<0.2$. For $S>0.2$, the data collected for a wind speed of 25 ms^{-1} was used. It was assumed that the spin rate remained constant throughout the trajectory.

Fig. 10a plots the trajectories of the two balls for the typical first serve which is being simulated. These trajectory plots illustrate that the worn ball lands a further 0.5 m towards the baseline, compared with that for the new ball. This is partly due to the lower drag coefficient of this ball, but is predominantly due to the ball having a lower lift coefficient. Therefore, the worn ball does not 'dip' as much during flight. This would be unlikely to deceive the receiver but if both new and heavily worn balls were used in the same game, the server would find it more difficult to place the ball accurately in the desired location on the court, due to the variation between balls.

In Fig. 10b, the angle of the shot for the worn ball has been adjusted to 4.65° which ensures that both balls land at the same point on the court surface. This is a more realistic simulation of how a player would adapt his stroke to compensate for the change in trajectory caused by the lower values of C_D and C_L of the worn ball. The worn ball passes over the net with a height that is 35 mm lower than the height of the new ball. The trajectories in Figs. 10a and b illustrate how a player uses top spin to control the trajectory of the ball, as commented on by Brody [21]. The top spin helps the player to ensure that the ball clears the net with sufficient height to allow for a finite margin of error, but also causes the ball to dip and land before the service line. This is particularly important for high-speed serves when the player has a very small 'window of opportunity'. This term is used to define the range of angles that the ball may rebound from the racket, and land within the boundaries of the



(a)



(b)

Fig. 10. Predicted trajectory for the new and heavily worn balls which have different values for C_D/C_L. (a) Both balls projected with same velocity and angle and (b) both balls propelled with same velocity, and impact at same location on court. The inset shows a close-up view of the end of the trajectory ($t = 0.68$ s).

tennis court. The lower lift coefficient of the worn balls reduces the player's ability to utilise top spin to make the ball land in the desired location.

5. Conclusions

In this study, the aerodynamic properties of a range of new and used tennis balls have been measured by supporting the ball in the working section of a wind tunnel. The drag coefficient of eight different brands of tennis ball was obtained, for non-spinning balls. It was found that there was a very high dependency between C_D and Re at low values of Re ($80 \times 10^3 < Re < 130 \times 10^3$), with C_D reducing as Re was increased. This was accounted for by the fact that, as Re is increased, the fluff on a tennis ball will begin to lay down. For $Re > 130 \times 10^3$, the drag coefficient is approximately independent of *Re*. Furthermore, the C_D values for all the new balls fall within a range of approximately 5%. This is approximately the range of the repeatability of the experiment, and therefore it can not be concluded whether there is any real difference between each ball brand.

The same testing was conducted for a range of used tennis balls. It was found that the C_D of the heavily worn ball which was subjected to 1500 impacts was 0.04 lower than the new ball. Clearly, the worn ball will travel through the air faster than the new ball and will give the receiver fractionally less time to react to return the ball.

In the second section of this testing, the spinner unit was used to spin the ball at speeds of up to 3000 rpm. It was found that C_D increased with the spin coefficient (normalised spin rate), for all balls. This was assigned to the induced drag which is generated when a lift force is obtained. Furthermore, the increase in C_D can be accounted for by the associated increase in the fuzz drag which is experienced by the ball, when it spins, due to the fluff 'standing up'. It was also shown that C_D (and C_L) were not solely functions of the spin coefficient S, but were also dependent on *Re*. This appears to contradict the conclusions of previous studies on spinning tennis balls.

It was concluded that there is no significant difference in the drag and lift coefficients of all the new ball brands. However, it was found that the drag and lift coefficients of the spinning worn balls were dependent on the number of impacts that the ball was subjected to. It was found that the ball subjected to 1500 impacts had a C_D value that was between 0.03 and 0.06 lower than that of the new ball. Of the C_L values collected for the worn balls at a wind speed of $25\,\mathrm{ms}^{-1}$, and a spin coefficient of 0.1, those of a heavily worn tennis ball (1500 impacts) are considerably lower than those of a new ball. However, at higher spin rates, the C_L values of both the new and worn tennis balls were approximately equal. At the higher wind speed of $50\,\mathrm{ms}^{-1}$, the difference between the new and worn balls is much smaller. A trajectory plot was conducted to illustrate that the difference in C_L values of the new and used tennis balls would change the impact location of the tennis ball by approximately 400 mm, for an identical serve. A trajectory analysis showed that changes in C_L had a negligible effect on the reaction time for the players. Players use the effect of spin on the trajectory of the ball for control, and therefore they are unlikely to want to play with a ball which has lower C_L values. This would discourage manufacturers from producing new tennis balls, which had a surface similar to a worn ball and therefore travelled faster through the air, because it has been shown that these balls generally have lower C_L values.

This study has illustrated the use of a wind tunnel to obtain the aerodynamic properties of non-spinning and spinning tennis balls. It has been shown that all new tennis balls exhibit similar values of C_D and C_L for the range of wind speeds and spin rates tested in this study. The study has highlighted that worn tennis balls can exhibit very different aerodynamic properties to those of new balls. Although it is accepted that manufacturers will not sell worn balls, even the most heavily worn ball in this study (1500 impacts) still 'looks and feels' like a tennis ball. Therefore, it may be possible for the manufacturers to produce a cloth which has the same texture as a worn ball. This would in effect change the nature of the game.

Acknowledgements

Grateful thanks go to the International Tennis Federation for the financial and practical support given to this project. The affiliation of Dr. S.J. Haake as Technical Consultant to the ITF is acknowledged.

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